

## Example

- Define  $T_n$  as the sum of the first  $n$  odd integers:

$$T_n = 1 + 3 + \cdots + (2n - 1).$$

- Notice that

$$\begin{aligned} T_n + (2 + 4 + \cdots + 2n) &= 1 + 2 + 3 + \cdots + (2n - 1) + 2n \\ &= \frac{2n(2n + 1)}{2} = n(2n + 1) = 2n^2 + n \end{aligned}$$

- Furthermore

$$\begin{aligned} 2 + 4 + \cdots + 2n &= \sum_{k=1}^n 2k = 2 \sum_{k=1}^n k \\ &= 2 \cdot \frac{n(n + 1)}{2} = n(n + 1) = n^2 + n. \end{aligned}$$

- Therefore

$$T_n = (2n^2 + \cancel{n}) - (n^2 + n) = n^2.$$

## Example

- Define  $T_n$  as the sum of the first  $n$  odd integers:

$$T_n = 1 + 3 + \cdots + (2n - 1).$$

- Notice that

$$\begin{aligned} T_n + (2 + 4 + \cdots + 2n) &= 1 + 2 + 3 + \cdots + (2n - 1) + 2n \\ &= \frac{2n(2n + 1)}{2} = n(2n + 1) = 2n^2 + n \end{aligned}$$

- Furthermore

$$\begin{aligned} 2 + 4 + \cdots + 2n &= \sum_{k=1}^n 2k = 2 \sum_{k=1}^n k \\ &= 2 \cdot \frac{n(n + 1)}{2} = n(n + 1) = n^2 + n. \end{aligned}$$

- Therefore

$$T_n = (2n^2 + \cancel{n}) - (n^2 + n) = n^2.$$

## Example

- Define  $T_n$  as the sum of the first  $n$  odd integers:

$$T_n = 1 + 3 + \cdots + (2n - 1).$$

- Notice that

$$\begin{aligned} T_n + (2 + 4 + \cdots + 2n) &= 1 + 2 + 3 + \cdots + (2n - 1) + 2n \\ &= \frac{2n(2n + 1)}{2} = n(2n + 1) = 2n^2 + n \end{aligned}$$

- Furthermore

$$\begin{aligned} 2 + 4 + \cdots + 2n &= \sum_{k=1}^n 2k = 2 \sum_{k=1}^n k \\ &= 2 \cdot \frac{n(n + 1)}{2} = n(n + 1) = n^2 + n. \end{aligned}$$

- Therefore

$$T_n = (2n^2 + \cancel{n}) - (n^2 + n) = n^2.$$

## Example

- Define  $T_n$  as the sum of the first  $n$  odd integers:

$$T_n = 1 + 3 + \cdots + (2n - 1).$$

- Notice that

$$\begin{aligned} T_n + (2 + 4 + \cdots + 2n) &= 1 + 2 + 3 + \cdots + (2n - 1) + 2n \\ &= \frac{2n(2n + 1)}{2} = n(2n + 1) = 2n^2 + n \end{aligned}$$

- Furthermore

$$\begin{aligned} 2 + 4 + \cdots + 2n &= \sum_{k=1}^n 2k = 2 \sum_{k=1}^n k \\ &= 2 \cdot \frac{n(n + 1)}{2} = n(n + 1) = n^2 + n. \end{aligned}$$

- Therefore

$$T_n = (2n^2 + \cancel{n}) - (n^2 + n) = n^2.$$

## Example

- Define  $T_n$  as the sum of the first  $n$  odd integers:

$$T_n = 1 + 3 + \cdots + (2n - 1).$$

- Notice that

$$\begin{aligned} T_n + (2 + 4 + \cdots + 2n) &= 1 + 2 + 3 + \cdots + (2n - 1) + 2n \\ &= \frac{2n(2n + 1)}{2} = n(2n + 1) = 2n^2 + n \end{aligned}$$

- Furthermore

$$\begin{aligned} 2 + 4 + \cdots + 2n &= \sum_{k=1}^n 2k = 2 \sum_{k=1}^n k \\ &= 2 \cdot \frac{n(n + 1)}{2} = n(n + 1) = n^2 + n. \end{aligned}$$

- Therefore

$$T_n = (2n^2 + \cancel{n}) - (n^2 + \cancel{n}) = n^2.$$